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Determination of Kalman filter TRF solutions based on white noise station coordinate behavior

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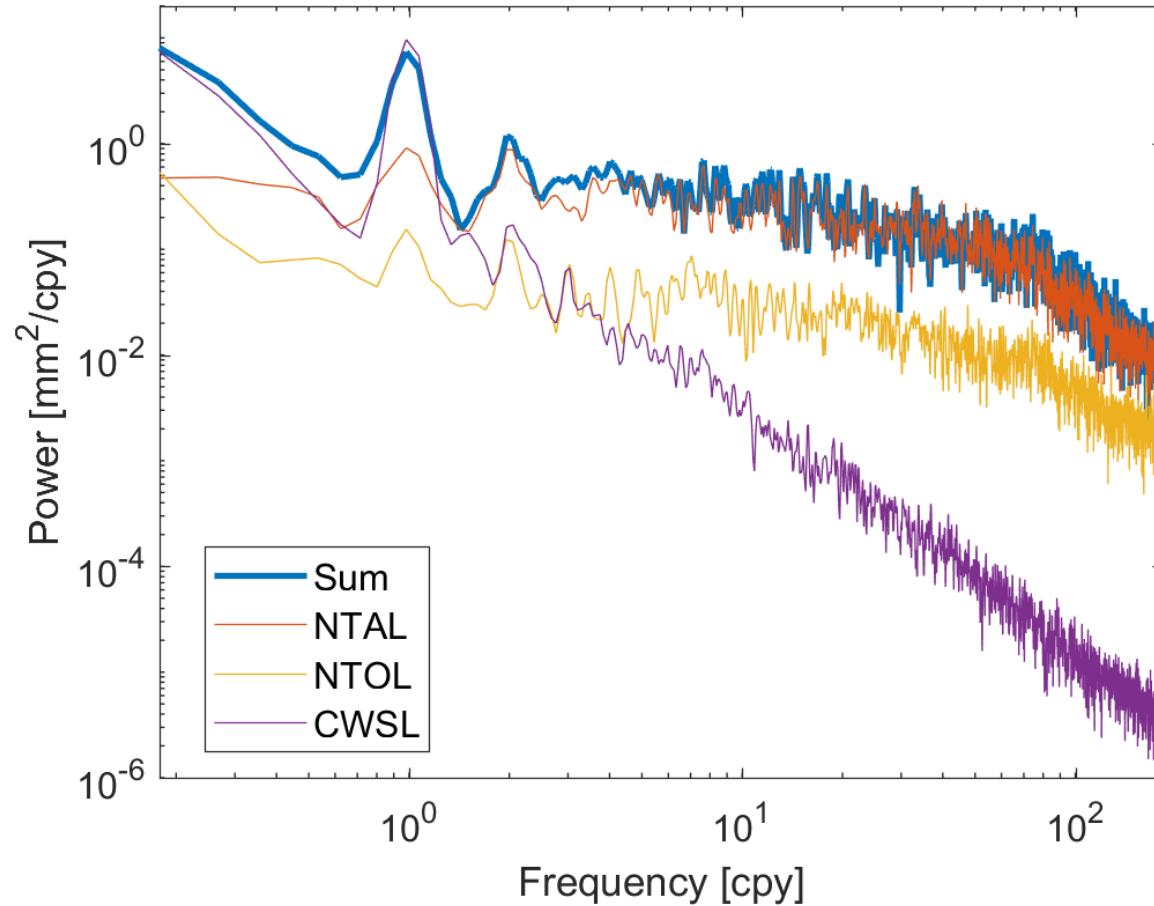


Introduction

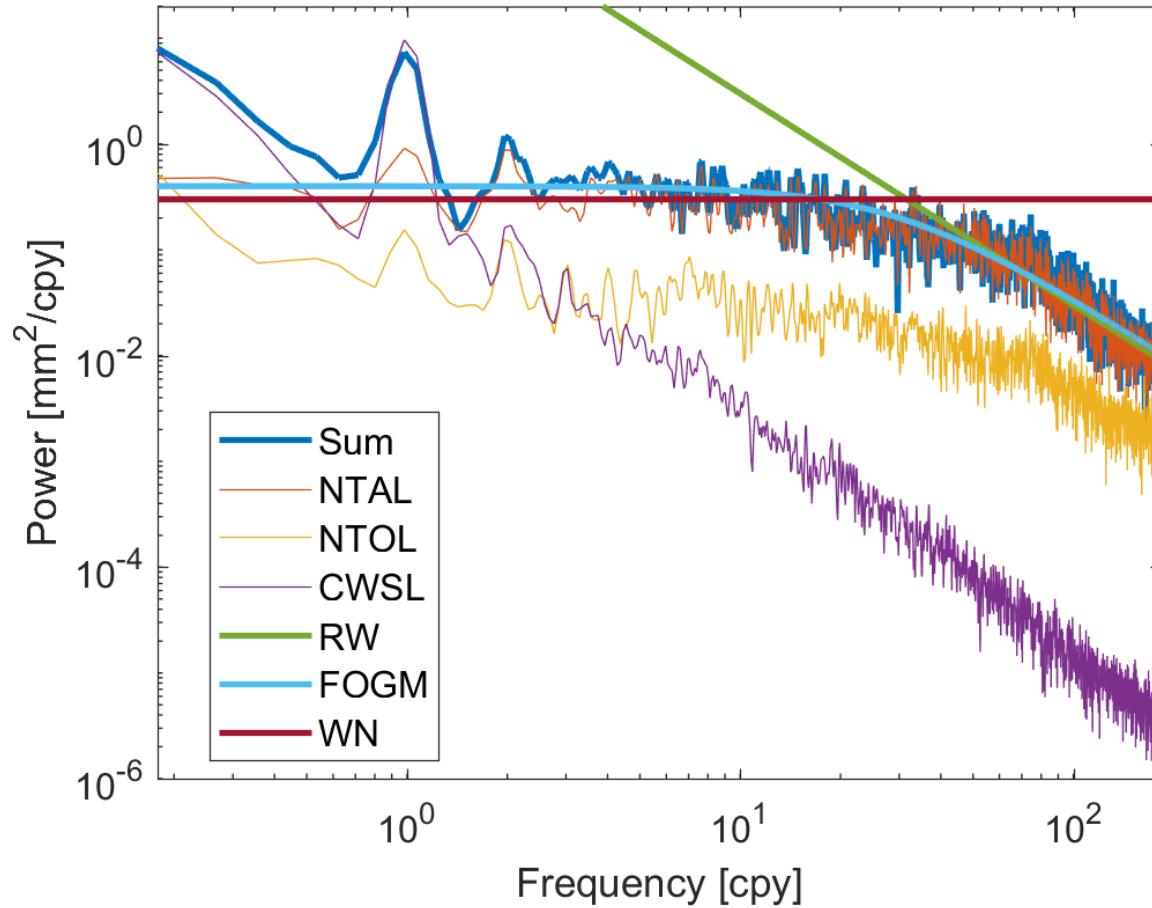
- **Kalman filtering** (KF) for **terrestrial reference frame** (TRF) determination: primary example JTRF2014 (Abbondanza et al., 2017)
- Usually based on the assumption that station coordinates behave like **random walks** (RW)
- This study: Kalman filter TRFs based on different stochastic assumptions: **white noise** (WN) and **first order Gauss-Markov** (FOGM)

Which stochastic process
should be used for KF TRFs?

Non-tidal loading displacements



Non-tidal loading displacements



Kalman filter formalism

State propagation equation: $\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{w}_k$

Previous approach:

$$\mathbf{F}_k = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix} \quad \mathbf{x}_k = \begin{pmatrix} x \\ v \end{pmatrix} \quad \mathbf{w}_k = \begin{pmatrix} w_k \\ 0 \end{pmatrix}$$

\mathbf{F}_k ... transition matrix

\mathbf{x}_k ... state vector

\mathbf{w}_k ... noise vector

x ...station coordinate offset

v ...station velocity

Kalman filter formalism

State propagation equation: $\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{w}_k$

Previous approach:

$$\mathbf{F}_k = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix} \quad \mathbf{x}_k = \begin{pmatrix} x \\ v \end{pmatrix} \quad \mathbf{w}_k = \begin{pmatrix} w_k \\ 0 \end{pmatrix}$$

New approach:

$$\mathbf{F}_k = \begin{pmatrix} 1 & \Delta t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varphi \end{pmatrix} \quad \mathbf{x}_k = \begin{pmatrix} x_{\text{offset}} \\ v \\ x_{\text{noise}} \end{pmatrix} \quad \mathbf{w}_k = \begin{pmatrix} 0 \\ 0 \\ w_k \end{pmatrix}$$

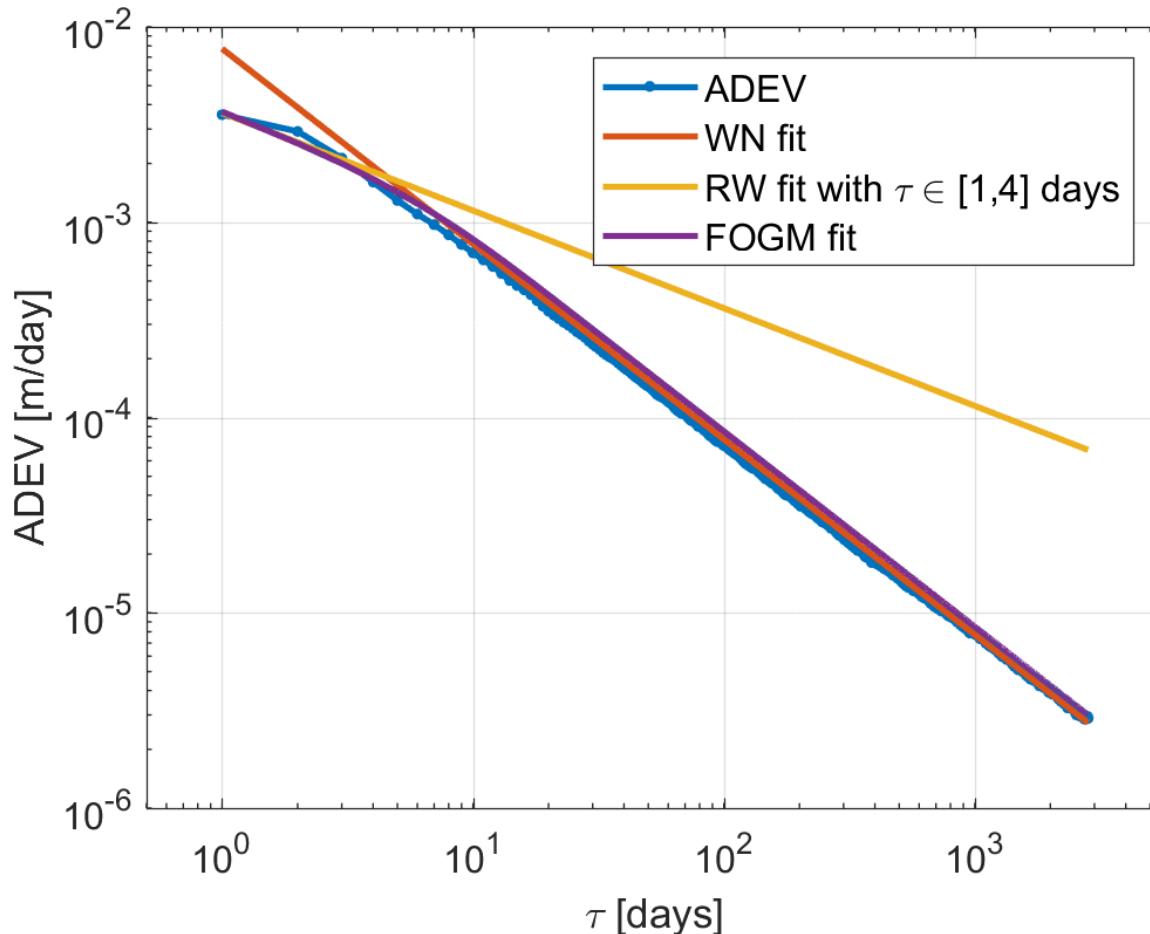
$$\varphi = e^{-\beta \Delta t}$$

WN: $\beta = \infty, \varphi = 0$

RW: $\beta = 0, \varphi = 1$

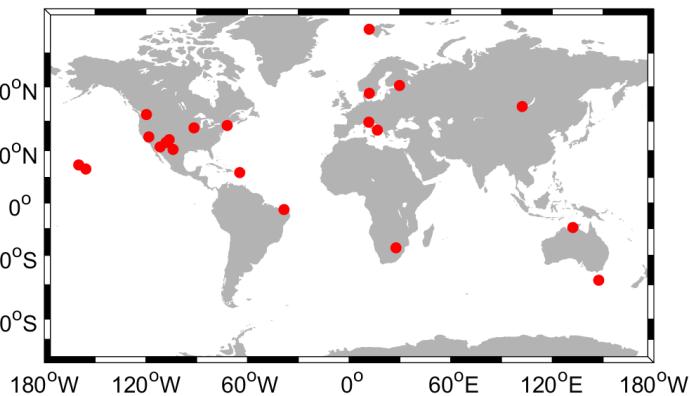
Estimation of noise parameters

... using Allan standard deviation (ADEV)

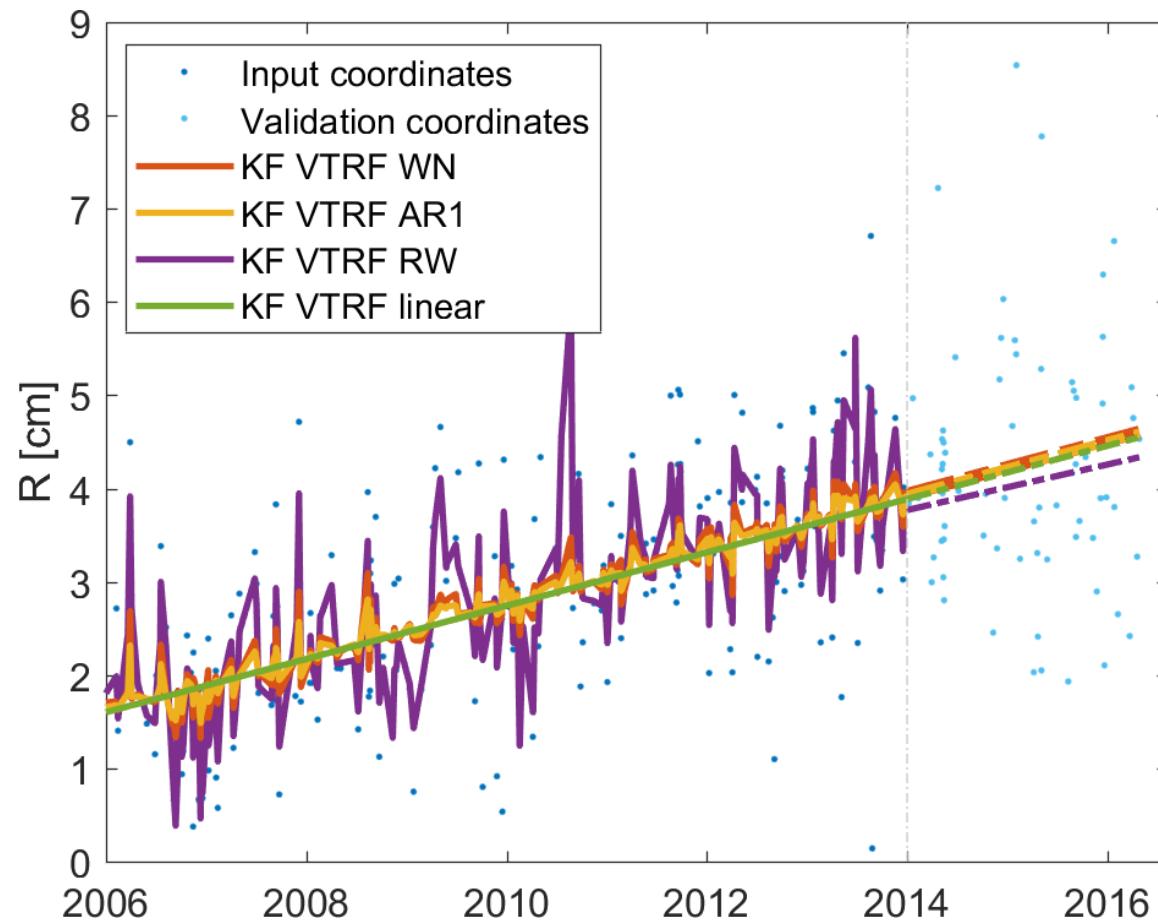


VLBI TRF solutions

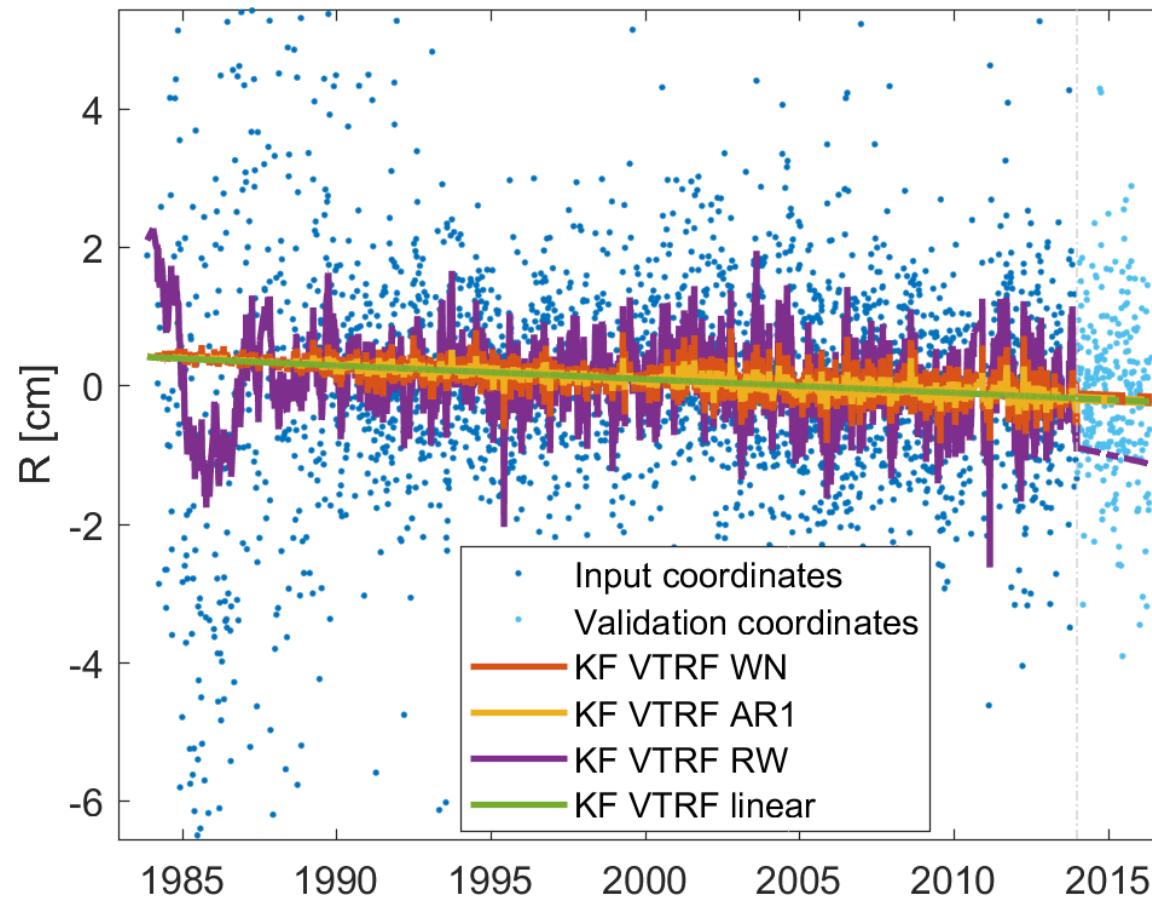
- ... based on different stochastic assumptions:
 - WN
 - FOGM / AR1
 - RW
 - Linear (zero noise)
- VLBI data between 1980 and 2014.0:
 - Computation of TRFs
 - Evaluation of TRF **precision**
- VLBI data between 2014 and 2016.5
 - Evaluation of TRF **predictions**



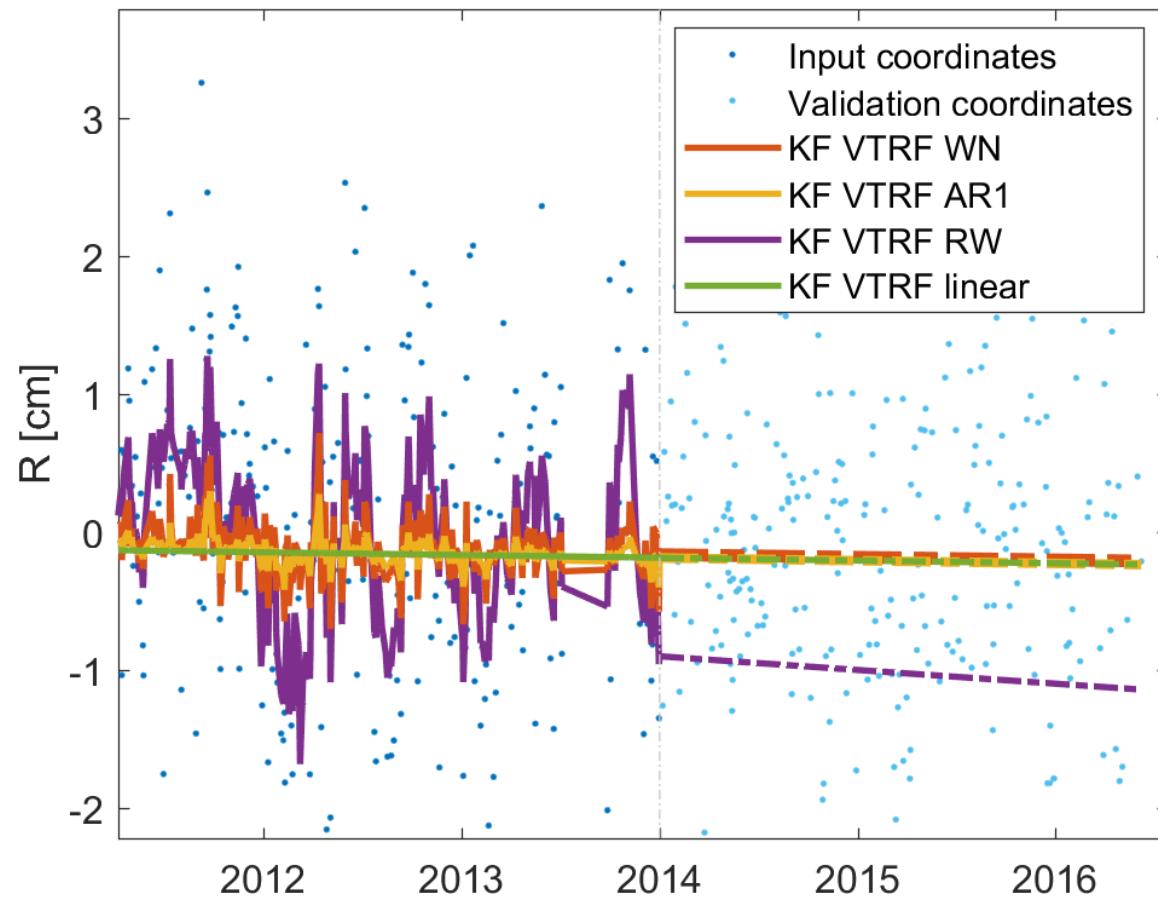
VTRF solution example: Onsala



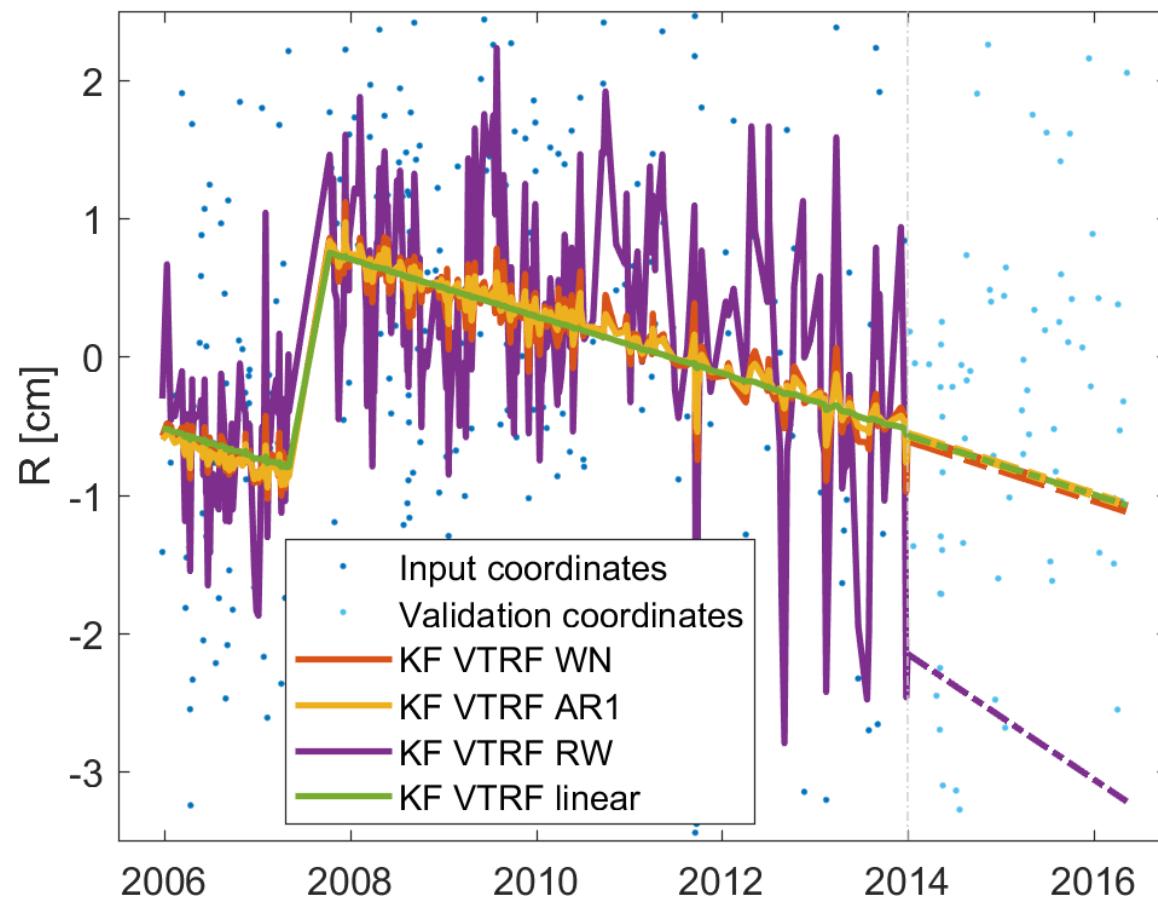
VTRF solution example: Wettzell



VTRF solution example: Wettzell



VTRF solution example: Zelenchukskaya



Difference of TRFs vs. VLBI coordinates

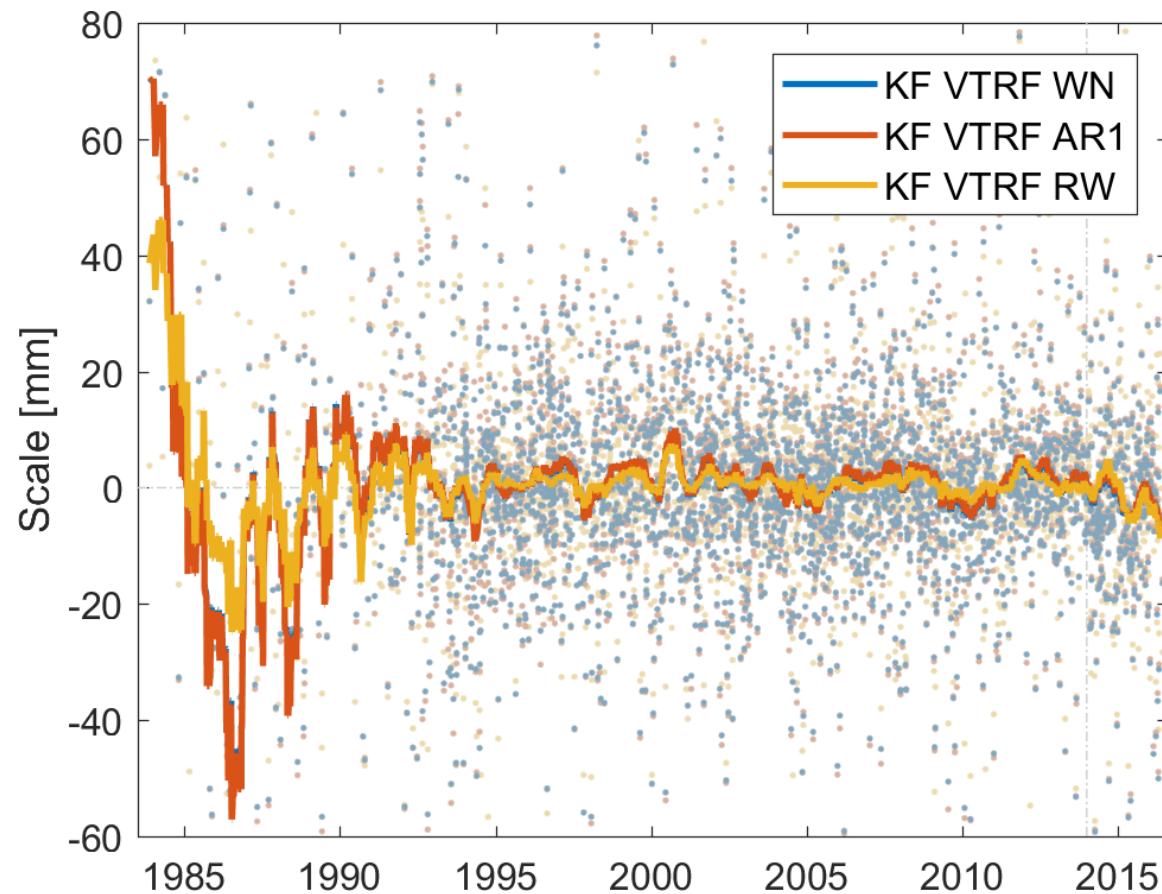
1980 – 2014.0

WRMS [mm]	R	E	N	3D
KF VTRF linear	11.4	3.9	5.4	14.0
KF VTRF WN	9.4	2.7	4.1	11.4
KF VTRF FOGM	8.8	2.6	3.8	10.7
KF VTRF RW	6.0	1.6	2.7	7.4

2014.0 – 2016.5

WRMS [mm]	R	E	N	3D
KF VTRF linear	13.2	4.1	5.1	15.1
KF VTRF WN	13.2	4.1	5.2	15.1
KF VTRF FOGM	13.1	4.1	5.3	15.0
KF VTRF RW	14.6	5.6	6.9	17.6

Scale of TRFs vs. VLBI coordinates



Scale of TRFs vs. VLBI coordinates

1980 – 2014.0

Scale [mm]	WM	WRMS
KF VTRF linear	0.4	9.0
KF VTRF WN	0.3	7.0
KF VTRF FOGM	0.2	6.9
KF VTRF RW	0.2	4.9

2014.0 – 2016.5

Scale [mm]	WM	WRMS
KF VTRF linear	-1.6	8.5
KF VTRF WN	-2.1	8.7
KF VTRF FOGM	-1.7	8.5
KF VTRF RW	-1.2	10.5

Scale of TRFs vs. VLBI coordinates

1980 – 2014.0

Scale [mm]	WM	WRMS
KF VTRF linear	0.4	9.0
KF VTRF WN	0.3	7.0
KF VTRF FOGM	0.2	6.9
KF VTRF RW 0.1	0.4	6.8

2014.0 – 2016.5

Scale [mm]	WM	WRMS
KF VTRF linear	-1.6	8.5
KF VTRF WN	-2.1	8.7
KF VTRF FOGM	-1.7	8.5
KF VTRF RW 0.1	-1.3	8.9

RW process noise
down-scaled by a
factor of **0.1**

Find out more:
Soja et al., JoG, 2018

Conclusions

- **Kalman filter TRF** solutions based on different stochastic processes
- **Random walk**
 - Best precision, weakest predictions
 - Can be tuned to produce reasonable predictions
- **First order Gauss-Markov and white noise**
 - Highest quality predictions, similar to linear solution
 - FOGM slightly more precise than WN
 - Require extra noise parameter for every coordinate component

Conclusions

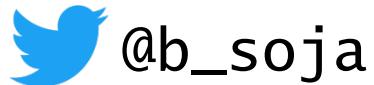
Which stochastic process
should be used for KF TRFs?

If computational costs are not a
concern, first order Gauss-Markov!

Otherwise, random walk!

Thanks for your attention!

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Acknowledgements

VLBI data: IVS (Nothnagel et al., 2015)

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References

- Soja et al., 2018: On the long-term stability of terrestrial reference frame solutions based on Kalman filtering. *Journal of Geodesy*, in press, doi: DOI: 10.1007/s00190-018-1160-0
- Abbondanza et al., 2017: JTRF2014, the JPL Kalman Filter and Smoother Realization of the International Terrestrial Reference System. *J. Geophys. Res. Solid Earth*, submitted.
- Nothnagel et al., 2015: The IVS data input to ITRF2014. International VLBI Service for Geodesy and Astrometry, GFZ Data Services. <http://doi.org/10.5880/GFZ.1.1.2015.002>

Backup slides

Determination of noise parameters

$$\sigma_y^2(\tau) = \frac{1}{2\tau^2} \langle (x^2(t + 2\tau) - 2x(t + \tau) + x(t))^2 \rangle$$

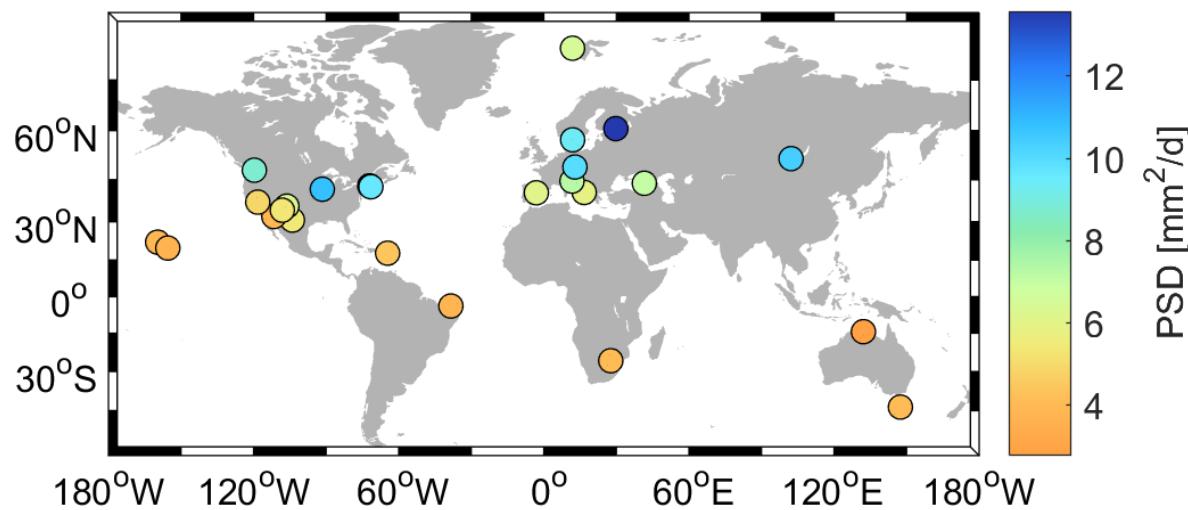
WN: $\sigma_y^2(\tau) = 3\Phi/\tau^2$

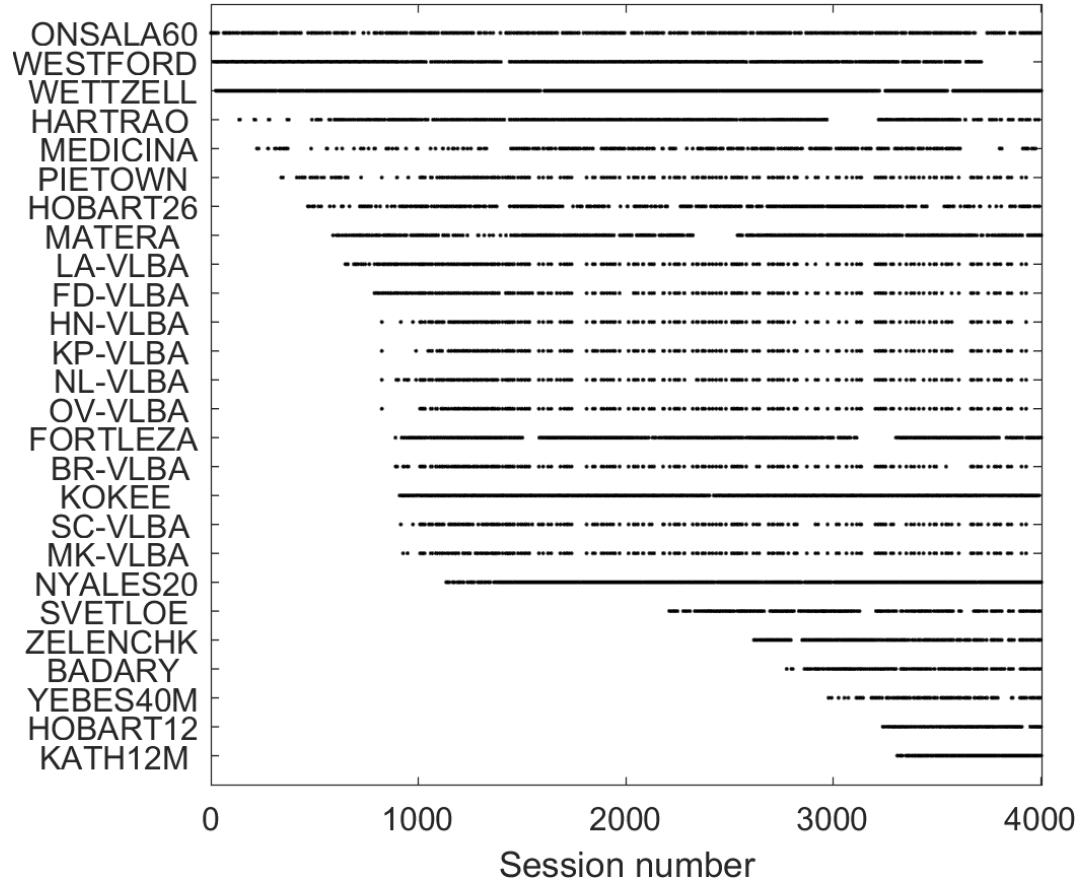
RW: $\sigma_y^2(\tau) = \Phi/\tau$

AR1: $\sigma_y^2(\tau) = \frac{\Phi}{2\beta} (3 - 4e^{-\beta|\tau|} + e^{-2\beta|\tau|})$

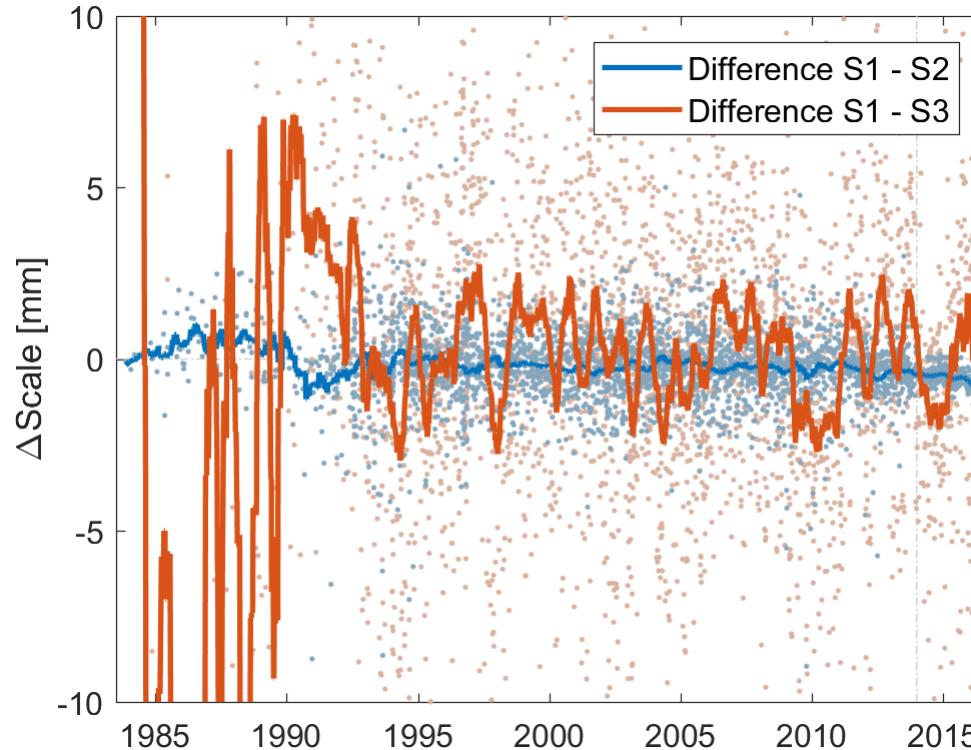
VLBI data

- Selection of **4006 global VLBI sessions**:
 - 3607 sessions between 1980 and 2014
 - - computation of the TRFs
 - - evaluation of TRF precision
 - 399 sessions between 2014 and 2016.5
 - - evaluation of TRF long-term stability
 - - computation of real time solutions
- Selection of **26 stations**:
 - At least 2.5 years of frequent observations before 2014
 - At least 0.5 year of frequent observations after 2014
 - No post-seismic displacements affecting predictions





Scale of TRFs vs. VLBI coordinates



Scale of TRFs vs. VLBI coordinates

1980 – 2014.0

Scale [mm]	WM	WRMS	Offset	Drift [mm/yr]
KF VTRF linear	0.4	9.0	1.0	0.06
KF VTRF WN	0.3	7.0	0.5	0.03
KF VTRF AR1	0.2	6.9	0.6	0.04
KF VTRF RW	0.2	4.9	0.2	0.00

Scale of TRFs vs. VLBI coordinates

2014.0 – 2016.5

Scale [mm]	WM	WRMS	Offset	Drift [mm/yr]
KF VTRF linear	-1.6	8.5	-2.7	1.00
KF VTRF WN	-2.1	8.7	-2.8	0.69
KF VTRF AR1	-1.7	8.5	-2.5	0.81
KF VTRF RW	-1.2	10.5	-1.5	0.25

WRMS of Helmert trafo residuals

1980 – 2014.0

WRMS [mm]	R	E	N	3D
KF VTRF linear	3.3	2.0	2.3	5.4
KF VTRF WN	2.2	1.2	1.4	3.5
KF VTRF AR1	2.1	1.2	1.4	3.4
KF VTRF RW	1.3	0.7	0.8	2.0

Difference of TRFs vs. VLBI coordinates

2014.0 – 2016.5

WRMS [mm]	R	E	N	3D
KF VTRF linear	7.6	3.1	3.8	10.4
KF VTRF WN	7.6	3.1	3.8	10.4
KF VTRF AR1	7.5	3.1	3.9	10.3
KF VTRF RW	8.2	3.7	5.0	12.2